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OF
DEMONSTRATIVE SCIENCE
CONSIDERED.

42

THE LIMITS

OF

DEMONSTRATIVE SCIENCE

CONSIDERED

IN A LETTER

TO THE

REV. WILLIAM WHEWELL, D.D.
MASTER OF TRINITY COLLEGE, CAMBRIDGE;
AUTHOR OF THE HISTORY AND PHILOSOPHY OF THE INDUCTIVE SCIENCES.

BY THE

REV. HENRY LONGUEVILLE MANSEL, B.D.
FELLOW AND TUTOR OF ST. JOHN'S COLLEGE, OXFORD;
AUTHOR OF PROLEGOMENA LOGICA.

OXFORD,
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St. John's College, Oxford,
April 12, 1853.

DEAR SIR,

I REGRET that I have been unable to pay earlier attention to the Letter which you did me the honour to address to me in September last. And even now, the remarks I am about to make on the subject of that Letter have been somewhat hastily drawn up, and must be considered rather as explanatory of previous statements than as embodying the results of subsequent study. But the subject is one in which I feel much interest; your own remarks have contributed considerably to simplify the question apparently at issue between us; and I am not without hopes that a few explanations on my part may help in some degree to settle a controversy which perhaps is more verbal than real.

Another motive I may be pardoned for confessing to have had some influence with me. "It is not often," observes a critic in the *Literary Gazette*, "that writers so distinguished as Dr. Whewell engage in controversy with those who criticise their published works." That the efforts of a tyro in philosophy should have attracted this notice from one of its most eminent masters is in itself most gratifying, while the candid and courteous tone of your Letter calls for a grateful acknowledgment on my part, and encourages me to hope that the observations which I have now to offer will be received in the same kind and liberal spirit.

Let me begin with a few words on the subject with which your Letter commences, the reception of the philosophy of Kant in this country. The disparaging criticism of Stewart must be taken for what it is worth, coupled with the author's confession of his entire ignorance of the German language, and his consequent reliance on translations and second-hand authorities. To this may be added the judgment of the greatest philosophical critic of the age, "that the tone and tenor of Mr. Stewart's remarks on the philosopher of Koenigsberg are remarkable exceptions to the usual cautious, candid and dignified character of his criticism[a]." The opinion of the "gentleman who has published a History of Moral Science," will hardly be deemed worthy of a serious notice.

It would probably astonish some of the critics who talk so comprehensively of German Metaphysics and German Theology, as if all Germans held the same opinions, to be told that the purport of the philosophy of Kant is to teach a lesson

[a] Sir W. Hamilton, *Reid's Works*, p. 886. n.

of humility, to inculcate the very limited nature of human faculties and human knowledge. And yet this is strictly true, however little of the same spirit may be discerned in the writings of Schelling or Hegel. Notwithstanding the historical connection between these earlier and later forms of German speculation, there is little real sympathy between the philosopher who announced as the moral of his critique, "Tecum habita, et noris, quam sit tibi curta supellex," and him who proclaimed his Logic as the "exhibition of God as he is in his eternal essence, before the creation of nature and of finite spirits." But Kant, great as has been his reputation, has not received his true honour in his own country. He is the philosophical offspring of Locke and Hume; his writings are the natural supplement and corrective of theirs; and it may be that the spirit of philosophy is not so extinct among the countrymen of Locke and Hume, but that the "unsightly root" of the German sage may yet bear in another soil the bright golden flower which it has failed to produce in its own[b].

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[b] Since the above remarks were written, I have met with the following judgment of a very competent critic, Mr. Morell, in his recently published "Elements of Psychology," p. 241. "There is a very prevalent opinion in this country that the writings of Kant are obscure and mystical. This opinion, I am bold to say, is wholly due either to the entire want of philosophical culture in the minds of popular writers who undertake to sit in judgment upon him; or to a positive

That this hope is not entirely fallacious, your own writings furnish no small assurance. It is to the honour of the Author of the *History and Philosophy of the Inductive Sciences* to have accomplished the most comprehensive and elaborate survey of the results of experience in a philosophical spirit the most remote from empiricism; to have acknowledged with Kant the existence of *à priori* conditions of consciousness in a work dedicated to the especial commemoration of the method of Bacon and the discoveries of Newton. It was mainly in consequence of my conviction of the very great services rendered by that work to Philosophy, that I ventured in the *Prolegomena Logica* to criticise a few passages, which, as understood by other readers as well as by myself, appeared to detract from the philosophical completeness of its theory. In the same

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ignorance of the meaning of the terms he employs. No one, I believe, who has taken the most moderate pains to read the works of this greatest of modern critics *intelligently*, will hesitate to agree with me in affirming that a more clear, steady, penetrating, dispassionate, *unmystical* mind, is not to be found in the whole circle of modern literature. His style is *incomparably* more lucid than that of Locke; his use of terms far more defined, and his meaning grasped, on the whole, by a less stretch of thought. All he demands (which is surely not very unreasonable) is, that you shall learn the force of his terms at starting, and then keep to their proper meaning throughout." I am happy to be able to adduce the high authority of Mr. Morell in support of the opinion which I have long held of the philosophical merits of Kant.

spirit I wish now to exhibit, as briefly and plainly as I can, what appears to be the present state of the controversy, as modified by your subsequent explanations.

The theory which refers all knowledge to the single source of Experience, I have always considered to be, under any explanation to which it has been subjected, untenable. I have objected indeed to the vague expression, *origin of ideas*, which tends to confound under one formula two very different questions, one concerning the formation of *concepts*, the other concerning that of *judgments*[c]. But as regards both concepts and judgments, and indeed the whole province of thought, pure and mixed, I have distinguished between the *matter*, which alone is given by experience, and the *form*, which is communicated by the mind itself[d]. In this sense, at least, I can heartily subscribe to the concluding words of your Letter. "And with regard to the image of vegetable developement, I may say, that as such developement implies both inherent forms in the living seed, and nutritive powers in earth and air; so the developement of our scientific ideas implies both a formative power, and materials acted on; and that, although the analogy must be very defective, we conceive that we best follow it by placing the formative power in the living mind, and in the external world the materials acted on: while the doctrine that all

[c] *Prolegomena Logica*, p. 157.

[d] Ibid. p. 228.

truth is derived from experience only, appears to reject altogether one of these elements, or to assert the two to be one."

It is not a valid defence of the opposite theory to use the term *Experience* in a wide sense, to include the consciousness of internal phenomena of mind, as well as of external phenomena of matter. It is not true that, even in this extended sense, all knowledge is derived from Experience. For Experience, whether internal or external, is of *individual* objects only, presented under the limiting conditions, of Space in the case of external phenomena, of Time in the case of both. My external experience may inform me of the presence of this or that colour or sound; my internal experience, of this or that appetite or volition; but in both cases the object is an *individual presented now or here,* and by these conditions distinguished from any similar phenomenon of consciousness presented in another place or at another time.

It thus appears that the very first step towards knowledge, the act of forming *general representative notions,* implies the existence of a faculty and a law of mind, instead of a mere *tabula rasa* on which experience registers its marks. In other words, the faculty of *comparison* by which the mind generalizes the results of external or internal presentation, is distinct from the consciousness of this or that single phenomenon, as the actual state of my mind at a given moment. Presentative Consciousness may be conceded to Experience. Representative Consciousness implies a formative power in the mind itself.

This point has been overlooked both by the advocates and by the opponents of Sensationalism. Mr. Lewes, for example, asserts that the "nisi ipse intellectus" of Leibnitz is merely Locke's doctrine expressed in an epigram[e]; in other words, that the Reflection of Locke contains all that Leibnitz added to Sensation. It may be so; but this would not prove Locke's theory to be complete, but Leibnitz's to be defective. *Reflection* must not stand for two things at once. If it is an internal sense, *presenting* individual phenomena of mind, it cannot at the same time be a process of thought *representing* all objects of experience, external as well as internal, by means of general notions. The truth is, that both theories are imperfect. That of Locke, though unfortunate in its language, points correctly enough to the two great sources of presentation, but omits the necessary cooperation of the representative faculties. That of Leibnitz, while rightly calling attention to the fact that necessary truths cannot be derived from experience alone, has not sufficiently discriminated between the two different sources of experience, internal and external, both distinct from the understanding itself.

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[e] *Biographical History of Philosophy*, vol. iii. p. 199.

The admission, therefore, of two distinct sources of knowledge, experience without and the mind within, is true as far as it goes, but admits of a further subdivision. The workings of the mind are of two kinds, so distinct from each other that they may be properly referred to two distinct faculties, the presentative, or source of internal intuition, and the representative, or source of thought. The former contributes the *matter* of one portion of our knowledge, that of the special phenomena of mind, and its facts will furnish the empirical basis of an inductive Psychology. The latter contributes the *form* of our whole knowledge, whether of ourselves or of the world without, and is the source of those concepts or general notions, under which the several phenomena of experience are reduced to specific unity.

But within these limits the formative power of the mind has no value as a criterion of truth and falsehood, or as a principle of distinction between necessary and contingent truths. The form of a concept or judgment is equally contributed by the mind, whether the concept represent a real or imaginary object, whether the judgment be true or false. Up to this point the operations of the mind are concerned only with the logical properties of thought, with the conceivability of a notion, or with the possible truth of a judgment. Before the same principle is admitted into a philosophy of physical science, it will be necessary to ask a further question. Does the mind itself, in any state of cultivation, furnish a valid criterion of the conformity of our notions and judgments to the actual phenomena of external things? Can certain ideas, once *clearly* and *distinctly* apprehended, enable us to place any physical science on a surer basis than that of observed facts; to determine, not merely empirically what the constitution of nature *is*, but axiomatically what it *must be?*

Perhaps there is a lurking ambiguity in the principal terms of this question. *Clearness* and *distinctness* were proposed by Descartes as criteria of the truth of ideas; but that philosopher has no where accurately distinguished between thought properly so called and other states of consciousness, nor between the *formal* clearness and distinctness which depend on the relation of one thought to another, and the *material* clearness and distinctness which depend on the relation of a thought to its object as presented. A concept is formally clear when it can be distinguished as a whole from any other; it is formally distinct when its several constituent elements can be analysed and distinguished from each other; but this is a criterion of logical reality alone, of the mental conceivability, not of the extra-mental existence of an object. If I have a clear and distinct notion of gold and of a mountain, I have also a clear and distinct notion of a golden mountain, through the objects of the two first notions are real, and that of the last imaginary. On the other hand, a concept will be materially clear and distinct if it accurately represents the character of the object itself and its component elements as they actually exist in nature. These qualities can obviously exist only in those notions which represent real objects; and in this case the clearness and distinctness can only be ascertained by an exact comparison of the notion with its object; i. e. by experience.

It is in the former sense only, that the terms *clearness* and *distinctness* are properly applicable to geometrical notions. It is not necessary that my conception of a straight line or a triangle should exactly conform to any material type. It is, indeed, by supposing such conformity to be necessary, that Mr. Mill attempts to reduce these concepts to mere generalizations from experience. He insists on "their capacity of being painted in the imagination with a distinctness equal to reality: in other words, the exact resemblance of our ideas of form to the sensations which suggest them[f]." This resemblance is so far from being implied in the mental distinctness of the geometrical idea, that it can only exist in the inverse ratio to it. In exact proportion to the clearness and distinctness of my conception of intelligible magnitudes, is my knowledge that they *do not resemble* the sensible. In proportion as I distinctly apprehend the mathematical character of a straight line or a triangle, in the same proportion am I conscious that no visible line is exactly straight and no visible body perfectly triangular. But if we admitted Mr. Mill's premises, I do not think we could avoid his conclusion. If there is any truth in the prevalent opinion that mathematical science is more certain than physical, it is precisely because in the former there is not the same need of comparing our ideas with our sensations; because sensible magnitudes are not the proper objects of geometrical reasoning.

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[f] Mill's *Logic*, vol. i. p. 309.

If, therefore, a philosopher asserts that physical science may attain to the same degree of certainty as mathematical, *provided only that its ideas can be apprehended with the same clearness and distinctness*, he would naturally be interpreted to mean, not material clearness and distinctness, which mathematical ideas neither have nor require, but formal clearness and distinctness, which they have. He would also be naturally understood to imply that it is entirely to this clearness and distinctness that the necessity of mathematical demonstrations is owing. I do not say that this interpretation could be consistently carried out by a reader of the *Philosophy of the Inductive Sciences:* on the contrary, he would find himself in that work continually reminded of the necessity of facts as well as of ideas, of material as well as of formal clearness and distinctness; but I say that such an interpretation is required to make the parallel between mathematical and physical science perfect, and I use it for the present only provisionally, to illustrate the difference between them.

Neither of the above positions is, I think, tenable; but in order to examine this question, we must first determine what is meant by *necessary truth.* There is a relative as well as an absolute necessity. An idea, however empirical in its origin, may logically involve certain consequences, which are so far necessary that they cannot be consistently denied of any object, so long as the original idea is affirmed of it. In this sense, any science may contain abundance of necessary and *a priori* principles, *its fundamental idea being once given*[g]. Such principles, however, do not contribute to the extension of our knowledge, but only to the more distinct consciousness of what we already possess. They exhibit separately the different attributes already comprehended in the given concept, but they do not enable us to add new ones. In the language of Kant, they are *analytical,* not *synthetical* judgments, and their necessity arises from the logical laws of thought, applied to the given matter of the fundamental idea.

This necessity is the only one that can arise from the formal clearness and distinctness of an idea; and the possible existence of such a necessity in physical science need not be disputed by the warmest advocate of empiricism; for no one will deny that a complex idea once given may be logically analysed, and its virtual contents actually exhibited. Nor is it denied that this power of analysis, depending mainly on the clearness and distinctness with which the fundamental idea is apprehended, will vary according to the intellectual and scientific culture of different persons. For though the laws which govern the process are logical only, and in their abstract character common to all men, yet the possession of the matter upon which they must operate in every actual exercise of thought, and the consequent possibility of their application in special cases, may be the privilege only of a small number of cultivated minds.

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[g] *Prolegomena Logica,* p. 92.

Thus far, all sciences may conceivably stand on an equal footing, though the conception has been much earlier realized in some instances than in others, and in many remains to this day only an idea of possible perfection which future generations may carry out. All sciences may contain axioms and deductions logically necessary as the legitimate developements of their fundamental ideas, and materially true *upon the assumption that the fundamental idea accurately represents the corresponding fact as it exists in nature.* Unless this assumption can be verified, they are necessary as *thoughts* only, and their claim to be regarded as *necessary truths* will depend upon the answer to the question, How is this fundamental assumption to be verified?

It is precisely at this point that the parallel between mathematical and physical science ceases. A science can be said in the highest sense of the term to contain *necessary truths*, if the object which its fundamental idea represents, as well as that idea itself, is given *a priori* by the laws or conditions of the mind itself, and not *a posteriori* in the facts of experience; in other words, if the matter as well as the form of its conception is necessary. A physical science will attain to mathematical certainty, if the verification of its fundamental idea, that is to say, the evidence of its correspondence to the phenomena of nature, can be placed on a basis equally necessary with the logical analysis of the idea itself. If, for instance, I can not only say, "the fundamental idea of Force being such as I conceive it, it is impossible to suppose the scientific consequences to be different from those which I have deduced;" but also, "it is impossible to conceive a world in which Force could be manifested otherwise than in conformity to my fundamental idea," Mechanics will then be elevated into a science of necessary truths. But if I can only say, "Force, as manifested in nature, does as a matter of fact conform to my fundamental idea, but it is conceivable that the world might have been constituted otherwise," the science may then be accepted as a true explanation of nature *as it is*, but not as a necessary demonstration of nature *as it must be*.

The latter is obviously the condition of any science in which the verification of the fundamental idea depends upon experience. If the accuracy of my fundamental idea of Force is questioned, I cannot vindicate it simply by pointing out the logical harmony of its consequences. I must appeal to experience to shew that the phenomena of force, as manifested in nature, are at least not inconsistent with the manner in which my idea represents them. I must occasionally verify my results as well as my principles by the same criterion, shewing that my scientific theory sufficiently accounts for the observed character of the natural phenomena; that the effects which are exhibited in fact correspond with the effects which must necessarily be exhibited, if my fundamental idea is true. Whether any existing science as a matter of fact can attain even to this degree of necessity; whether the developement of the clearest fundamental ideas does not continually require the assistance of further empirical premises, is another question; but the above is surely the utmost conceivable excellence in which a sober-minded physical philosopher can hope to exhibit that department of science: viz. *as a me-*

thodized series of the necessary consequences of a fundamental idea, verified by their conformity to the actual phenomena of nature as given in experience.

I say, a sober-minded philosopher; for a very different conception of natural philosophy has found favour with a people among whose intellectual merits sobriety of thought will hardly be reckoned. Philosophy, to be worthy of the name, is conceived as a *creative idea*, which, by a purely intellectual process, shall produce or reproduce the constitution of the world as existing *a priori* in the divine mind, and evolve an universe out of nothing. Such a conception, if realized, might indeed proclaim itself as a demonstrative exhibition of nature as it must be; but to this theory of physical necessity the whole spirit of the *Philosophy of the Inductive Sciences* is diametrically opposed. Nor is such a conception, supposing it possible, really entitled to be considered as a more perfect method of physical science. As I have elsewhere observed[h], a demonstration of a fact, if such were possible, would not add one tittle to the evidence of the fact as such in the eyes of any but an Egoist. By him it would be accepted as an additional proof that what are commonly considered as phenomena of the *non-ego* are really only modifications of mind and governed solely by mental laws. But to the Realist, who believes in the essential distinction of mind and matter, it could at most only suggest the possibility of a pre-established harmony between the laws of each, a suggestion which would require, in every special case, to be verified by an empirical examination.

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[h] *Prolegomena Logica*, p. 293.

Up to this point, then, the *necessity* of any physical statement, i. e. the *inconceivability* of its contradictory, is a formal necessity only, derived from the analysis of its fundamental idea, and belongs to the deductive, not to the inductive portion of the science. The synthetical judgments of Physics, those which state the relation of our ideas to the facts of nature, and those which add to an idea any new attribute beyond what it contained before, are based on an observation of facts as they are, derive their scientific value from their conformity to those facts, and are thus verified empirically. On the character of these latter judgments depends the whole question of the essential difference between mathematical and physical reasoning. If, as Mr. Mill maintains, mathematical ideas, like physical, are generalized from the phenomena of sense and must ultimately be verified by their agreement with those phenomena, then in mathematical as well as in physical science the synthetical judgments are *a posteriori*, and its reasonings are reduced to a statement of what *is*, instead of a demonstration of what *must be.* If, on the other hand, physical science could be shewn to gain its synthetical judgments *a priori*, it would be elevated to the grade of mathematical certainty, and the constitution of the world might be deduced from the laws and conditions of the human mind.

The truth I believe to lie between these two extremes. The synthetical judgments of Mathematics are *a priori,* as concerned with the subjective conditions of intuition, (Space and Time,) which it is impossible to annihilate or change in thought. The synthetical judgments of Physics are *a posteriori,* as concerned with the objective facts of intuition, which it is always possible to annihilate or change in thought, however constant may be our experience of the facts themselves. It was on the ground of this distinction that I observed in the *Prolegomena Logica,* "The clearness and distinctness of any conception can only enable us more accurately to unfold the virtual contents of the concept itself; it cannot enable us to add *a priori* any new attribute. In other words, the increased clearness and distinctness of a conception may enable us to multiply to any extent our analytical judgments; but cannot add a single synthetical one. Without something more than this, the philosopher has failed to meet the touchstone of the Kantian question, *How are synthetical judgments a priori possible?*"

In this passage, the terms *clearness* and *distinctness* were used in the formal sense, in which alone they are properly applicable to mathematical notions. In this sense they can give rise to analytical judgments only. On the other hand, if these terms be used in a material sense, to imply exact conformity with facts, they will give rise to synthetical judgments, but not *a priori.* To the *truth* of a science this is sufficient, but not to its *necessity.* If physical ideas could give rise to judgments equally necessary with mathematical, we should not only know the system of nature as it is, but also that no other system can be conceived as possible. It is not enough that no other system should be possible, so long as our fundamental idea remains the same, or so long as the laws of nature continue unchanged. We should also know it to be absolutely inconceivable that the present system could be destroyed and another substituted in its place, so as to give rise to a different fundamental idea, expressive of a different experience. We should believe not only that our fundamental idea correctly represents the existing phenomena of the world in which we live, but also that there cannot be in any other part of the universe, or at any other period of time, a system of phenomena other than our fundamental idea supposes.

All this is implied in geometrical necessity; and the explanation is to be found in the relation of its synthetical judgments to Space as a *subjective condition of intuition.* I do not understand by this expression the same thing that is meant in the *Philosophy of the Inductive Sciences* by a *funda-*

mental idea. And I lay stress upon this distinction, inasmuch as in the *Letter to the Author of Prolegomena Logica,* these two terms are spoken of as convertible. A fundamental idea is suggested by, if not altogether derived from, the facts of experience, and generalized by the mind from and after the observation of such facts. Hence its logical character is that of a *general concept* predicable of many individual objects, and its scientific value depends on the accuracy with which it represents those objects and explains the actual phenomena in each individual case. Now a concept is not limited by the laws of thought to any definite number of individuals, nor even to really existing individuals at all. Hence there is no difficulty in conceiving the destruction of any one of the individuals of which the concept is predicable, and consequently of all. My conception of a rose, for example, may have been drawn from the examination of ten actual roses, but does not in itself imply the existence of ten roses, neither more nor less. The notion once gained would remain if the number were reduced to nine, to eight, and finally to none at all. In such a case, the logical value of the concept as the representative of possible objects would remain unchanged, but its material value as the representative of actual objects is destroyed. On the other hand, a subjective condition, or, in Kant's language, *a form* of intuition, is not generalized by the mind from or after the facts of experience, for without it experience itself (and not merely a philosophy of experience) is impossible. Hence it is not a general concept, is not predicable or representative of a plurality of individuals, is not capable of verification by facts, and cannot be destroyed by any act of thought.

As an example, we may compare together the ideas of Force and Space. According to the mechanical conception of Force, as a cause of motion, known and measured by its effects, there are many forces in nature, greater and less, acting singly or in combination, and giving rise to a variety of phenomena. Any one of these may be conceived to be removed or changed with a corresponding change of phenomena; and there is no impossibility in supposing the existence of a world in which no exertion of force is manifested[1]. Force again is manifested in a particular class of sensible objects; and the objects may be observed by an unscientific mind without giving rise to the corresponding idea. On the other hand, Space is presented to consciousness as one only (for the distinction of spaces is purely arbitrary): it necessarily accompanies every perception and every imagination of sensible objects: it presents itself to every man's mind, and cannot in any act of thought be conceived as non-existent. A false conception of Force may vitiate our whole theory of the objects in which it is manifested; for Force is a representative concept, regulating our thoughts. A false conception of Space has no such consequences; for Space is a presented condition regulating our intuitions. Space, according to Locke's theory, as an idea derived from sensation, bears some resemblance to the fundamental ideas of science; but in this case it cannot be regarded as a form of intuition, and the theory fails to account for its principal characteristics. Space, according to Kant's theory, as a form of intuition, differs in a remarkable degree from the ideas of science. But the demonstrations of Geometry are not affected by this metaphysical controversy.

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[1] It may be true that the best mechanical conception of rest is that which represents it as produced by the equilibrium of forces; but this presupposes the idea of force as derived from motion. In a world wholly at rest, the conception of force would not arise.

For these reasons, I do not think that Geometry can in strict accuracy be described as founded on the idea of Space, but rather on that of *Magnitude*, or continuous quantity, which can only be conceived as in space. Magnitude, like Force, is a concept predicable of various subordinate objects, equal or unequal, like or unlike; and the position of the one in Geometry is in many respects analogous to that of the other in Mechanics. Force is the subject of the *common notions* or analytical principles employed in Mechanics, as Magnitude is of those employed in Geometry. For instance: that two equal and opposite forces will destroy each other's effects, is an analytical judgment implied in the notion of equal forces; as that if equal magnitudes are added to equals the sums are equal, is an analytical judgment implied in the notion of equal magnitudes. Again, Force as estimated by its effects may be divided according to the objects on which it acts, as on a particle, a rigid body, or a system: these correspond to the various subdivisions of magnitude, the straight line, the triangle, &c. A person with an erroneous conception of Force may be unable to see the cogency of the reasonings in Mechanics; and it is not impossible that one with an erroneous conception of Magnitude may not be able to follow the demonstrations of Euclid.

Nevertheless, these two fundamental concepts exhibit differences no less remarkable than their resemblances. Force as conceived in Mechanics is representative of force as actually existing in nature; and the accuracy (or in a material sense the clearness and distinctness) of the conception depends on its agreement with the fact. Magnitude as conceived in Geometry exists only in the imagination; and the mental image would acquire no new value in pure Geometry by its conformity (if such existed) to a physical fact. The value would be confined to applied Geometry alone. An erroneous conception of Force would vitiate the *matter* of mechanical reasoning, by introducing such attributes as are incompatible with the explanation of the phenomena, or omitting such as are essential. An erroneous conception of Magnitude would vitiate the *form* only of geometrical reasoning, by introducing attributes inconsistent with the subsequent deductions of the science. Our conception of Force, even if now true, may be conceived as becoming for mechanical purposes non-existent: i. e. the constitution of the world may conceivably be so changed that the conception may be no longer applicable. Our conception of Magnitude cannot be conceived as becoming geometrically non-existent; for it exists only as thought, and, being equally an object of thought in the conception of its non-existence, that conception is self-contradictory.

Neither Force in general nor Magnitude in general can give rise to synthetical judgments. For the subject of a synthetical judgment is not the concept as such, but the individual in which the conceived attributes are present. The judgment, that two straight lines cannot enclose a space, is not true if interpreted to mean " the notion of two straight lines is the notion of lines which cannot enclose a space:" the notion of two straight lines is the notion of two lines which lie evenly between their points, or whatever other definition we may adopt, and it is nothing else. So the judgment, that the pressure of two equal weights upon the fulcrum of a lever is not affected by the length of the lever, in its synthetical interpretation, (for it admits of more than one,) does not state the identity of the two notions as such, but a fact concerning the things signified. But the general concept as such cannot be depicted to sense or imagination as an individual. I cannot perceive or imagine magnitude or force in general, but only this particular magnitude or force presented now and here, as a thing without the mind or an image within. Hence, to give rise to a synthetical judgment, the fundamental idea must be *individualized.* And in this process we find the same differences between the two sciences as in the concepts themselves. The synthetical judgments of Geometry are derived from the *imagination of a possible object* corresponding to a given concept: those of Mechanics from the *perception of an actual one.* I depict in my imagination a pair of straight lines, and the image as presented does not enclose a space; but the mind in this case is governed only by its own law of individualizing a concept, and consequently the image presented contains no adventitious elements, and is an adequate representative of any other image of two straight lines which I may depict on any future occasion. To conceive otherwise, I must conceive my mind as governed by other laws than its own, which, as conception is itself the servant of the law, is impossible. On the other hand, the individual instance which I perceive of a rigid body acted upon by forces may contain many accidental features not common to other instances. There is need of experience and comparison of various instances to reject what is accidental and retain what is essential; and even then the essential features, being only such as the laws of nature require, may still be annihilated in thought; for thought is not bound by the laws of force acting on bodies.

To exhibit this in special instances, it will be necessary to separate, in the axioms of Mechanics, that which is logically necessary as an analysis of notions from that which is empirically true as a statement of facts; for these are frequently combined in the same formula. Take, for example, the first axiom of Statics as given in the *Mechanical Euclid.* "If two equal forces act perpendicularly at the extremity of equal arms of a straight lever to turn it opposite ways, they will keep each other in equilibrium." "Axioms of this kind," we are told, cannot be verified by experiment, "without assuming other principles, which require as much proof as these do. If, for instance, Archimedes had wished to ascertain by trial whether two equal weights at the equal arms of a lever would balance each other, how could he know that the weights *were* equal by any more simple criterion than that they *did* balance?" In one sense the above axiom is a mere identical judgment. If by *weight* we understand a force manifested by its effects, and the equality of the weights is ascertained by their equilibrium on the lever, it is a mere identical judgment to say that equal (or balancing) weights balance each other. To make the axiom synthetical, *weight* must be interpreted to mean *heavy body;* and it is assumed that bodies equally heavy on one occasion will be equally heavy on any other. This may be true as a fact in nature, but surely the contradictory is not *inconceivable.* It is not inconceivable that the law of gravitation may be suspended altogether, or that bodies equally heavy to-day in England may be unequal to-morrow in France[k]. The same criticism may be applied to the confirmation of the axiom from the principle of sufficient reason. Assuming the uniform action of gravity, it furnishes a sufficient reason for the uniformity of the result; but if we can conceive this law to be changeable, its change will be a sufficient reason for any uncertainty in phenomena.

The other axioms of Mechanics are I believe open to similar objections. The third, that the pressure on the fulcrum of a lever is equal to the sum of the weights, has been criticised by Sir William Hamilton on a principle substantially the same as that which I employed in the *Prolegomena Logica* in reference to the first. I cannot help regarding this coincidence of two arguments published nearly at the same time and independently of each other, as some confirmation of the truth of the criticism. If by the sum of the weights is meant the sum at *any one* distance from the fulcrum, they are not *equal to*, they *are*, the pressure; and so far the judgment is merely identical. If, on the other hand, we mean the sum of the weights at *all distances* from the fulcrum, this assumes the uniform action of gravity, and reduces the axiom to a result of experience. It would not be difficult to apply the same reasoning to other cases. But I fear my argument is already getting somewhat prolix.

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[k] This criticism has been stated in another form in *Prolegomena Logica,* p. 292.

There is one other point of contrast, however, between Geometry and Mechanics, which it may be as well to notice: I mean the effect produced on the two sciences by a change in the fundamental idea. It is urged, that "to a person who is only beginning to think geometrically, there may appear nothing absurd in the supposition that two straight lines may enclose a space. And in like manner to a person who is only beginning to think of mechanical truths, it may not appear to be absurd that in mechanical processes Reaction should be greater or less than Action." The parallel seems to me to be more apparent than real. If two persons have different conceptions of force, both are designed to be representations of the same fact. Both persons mean by force, the cause of that motion which both alike perceive in bodies: they differ only as having a more or less adequate conception of the manner and conditions of its operation. Hence they may properly be said to have two ideas of the same thing, differing in accuracy or completeness. But if the image presented to my mind in connection with the name *straight line* corresponds to that suggested to another man's mind by the name *curve,* we are not thinking of the same thing; for the thing is the image, and nothing but the image. We differ only in an equivocal use of language. I have not a wrong conception of a straight line, for I am not really thinking of a straight line at all: I am only expressing a valid thought in wrong language. My conception is really of a curved line; and the thought, as such, is mathematically accurate; but I am wrong in giving it the name which geometers have attached to a different image. And this explains the ground of a distinction which I have already noticed; viz. that an erroneous conception in Mechanics vitiates the matter of the reasoning, while an erroneous conception (if we may call it so) in Geometry vitiates only the form. The matter of Geometry exists only as a thought, and is produced by the mind acting according to its own necessary laws in the individualization of concepts. The attributes forming the concept being once given, error in the thought as such is impossible; for the mental process is amenable to its own laws only, and these it necessarily obeys in being an act of thought at all. But the given attributes themselves on any occasion may not be the same as those associated with the same name by the author of a particular geometrical demonstration. Hence it may be that, while apparently following his reasoning, I really start from a different fundamental thought, comprehending attributes incompatible with his demonstration. Neither my thought nor his is erroneous *per se*, but a contradiction arises from the attempt to unite them into one concept under one name. But in Mechanics two different conceptions are amenable to a test by which they can be distinguished as true or false; viz. the external fact which each is supposed to represent.

On these grounds it has been a matter of controversy among philosophers, whether mathematical science can properly be said to contain necessary *truths*. That they contain necessary *thoughts* is unquestionable; but can those thoughts be properly called *true?* The question may be answered in the affirmative or negative, according to our definition of truth. If truth be defined as consisting in the agreement of our thoughts with things out of the mind, Mathematics cannot be said to contain truths. Locke's defence of mathematical truth[1] is, I think, untenable, and inconsistent with other parts of his philosophy. Without previously recognising the distinction between analytical and synthetical judgments, and the *a priori* possibility of the latter, there is no more truth in the properties considered as belonging to the idea of a circle or a rectangle, than in those considered as belonging to the idea of a centaur or a chimera. Nor can we take the ground that the properties of the triangle are true of the real figure, if such exists. For this assertion can only be made on the ground of some previous relation of truth in the ideas themselves. But the above definition of truth is objectionable. Truth does not consist in the agreement of thoughts with *things in themselves,* but with *intuitions.* The thing is known only as presented in some intuition or other, whether *a priori,* as the image in Mathematics, or empirically, as the object of a sensible perception. The justification of the expression *necessary truth,* in relation to Mathematics, is to be found in the fact, that the concept proceeding by its own laws necessarily embodies itself in the corresponding image, which as a modification of space is intuitively invested with additional attributes. But the necessity of truth implies the impossibility of the corresponding falsehood; and it is therefore incorrect to speak of a false conception in Mathematics; for either it is no conception at all, or its error is one of language only. The opinion of Stewart, who regards mathematical reasonings as containing hypothetical truth, based on the assumption of the definitions, confounds mathematical necessity with logical, and overlooks the distinction between synthetical and analytical judgments. But it is unnecessary to dwell upon an error which has been fully refuted in Dr. Whewell's *Essay on Mathematical Reasoning*.

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[1] *Essay*, iv. 4. 6.

A word, in conclusion, in vindication of the consistency of my views on this subject with those maintained in other parts of the *Prolegomena Logica*. I have acknowledged, in addition to logical and mathematical necessity, a metaphysical necessity, which I consider as more properly psychological, and, in a wider sense of the term, a physical necessity. This last, however, is in relation to thought contingent only; its contradictory being always *conceivable*, though never actually *true*[m]. In this sense, I have never objected to the use of the term *necessary truths*, to express the results of physical science. As regards metaphysical necessity: the ideas of Substance and of Cause, when they are supposed to imply any thing more than mere phenomena, are so far from being treated as foundations of science, that they are regarded as, except in Psychology, essentially negative and obscure, and such as can never attain to scientific clearness and distinctness[n]. Physical causality, indeed, conceived according to Hume's limitation as an invariable relation of phenomena, is considered as the foundation of that belief in the uniformity of nature which renders physical science possible[o]. In this extent, it is admitted to be something more than a mere empirical generalization; being regulative of, not governed by, the facts of experience. But then, in this sense, it is to physical science no more than the principle of sufficient reason is to Logic. It does not discover any special law of nature: it only states the fact that all natural phenomena are governed by some law or other. It is thus equally compatible with the assignment of a true or a false cause in any special case; and it is equally presupposed in all attempts at scientific investigation, whether their theories be sound or unsound. The scientific discovery, that this particular effect is to be referred to this particular cause and to no other, remains to be made by experience; and when it is made, it possesses no higher degree of necessity than that of a fact in the constitution of nature, which is thus, but might conceivably have been otherwise.

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[m] *Prolegomena Logica*, p. 162.

[n] Ibid. p. 131, 140.

The principle of the permanence of substance, which is only another form of that of causality, may also be referred to a psychological origin[p], and in its most general statement is a necessary principle, controlling and correcting all possible experience. But in this general form it is also compatible with true or false theories in chemistry or any other branch of physical science; and it does not enable us to ascertain *a priori* any particular law as governing this or that special combination of elements. But surely the very fact, that such principles are necessary conditions of thought, and have, as such, been universally adopted by philosophers in all ages, is itself an additional proof if proof were needed, that the actual constitution of nature is not determined by them as necessary. For how otherwise could various and contradictory opinions in matters of science have been held by men alike thinking under these regulative laws? How otherwise could a true conception of the constitution of nature as a whole consist with ignorance or error in relation to any of its details?

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[o] *Prolegomena Logica*, p. 145.

[p] See Sir W. Hamilton's *Discussions on Philosophy*, p. 585.

I have now got back again to the position which I originally maintained; viz. that no matter of fact, that is to say, no actual phenomenon of external nature, can in any possible state of human knowledge be a matter of demonstration. And it is this principle that fixes the limits of demonstrative science, separating the results of the necessary laws of mind from those of the generalized phenomena of matter. This position I hold to be psychologically certain, as the consequence of the distinction between the internal laws of thought and intuition, which may be passively suspended by ceasing to think at all, but which cannot be actively transgressed, and the external laws of natural phenomena, which may furnish a sufficient reason why certain phenomena take place in a certain manner, but which furnish no reason at all why I must think so. If this distinction is properly pointed out, I do not think that the term *necessary truths* is in itself objectionable as applied to the physical sciences. But the language is likely to mislead, if the difference between the various kinds of necessity is represented as one of degree only, and dependent upon accidental and remediable deficiencies, so that a physical idea may possibly be apprehended as clearly and distinctly as a mathematical one, and the science be raised to an equal grade of necessity. It is not a question merely concerning the *grounds* of the different kinds of necessity[q]; for the difference of grounds gives rise to a corresponding difference in the mental conviction itself. This difference need not be acknowledged by the advocates of either of the extreme theories between which the *Philosophy of the Inductive Sciences* preserves in its conception and general spirit a just and philosophical mean. The sensationalist, who reduces mathematical science to a generalization from experience, may speak indifferently of *necessary* truths in Mathematics and in Physics; for both alike are conceived as attaining to the highest necessity of which the human mind is capable, a constant association of ideas, the result of uniform experience. The transcendentalist, who dreams of producing a world from his creative thought, may use the same language; for Nature is with him, like Mathematics, a function of thought alone: no stubborn facts are allowed to rebel against his theory; or if they do, so much the worse for the facts. It is precisely because the *History and Philosophy of the Inductive Sciences* falls into neither of these extravagancies, but adopts a middle course, more sober, more thoughtful, more comprehensive than either, that its readers are occasionally struck with the apparent inconsistency of language which seems properly to belong to a more extreme and less philosophical theory.

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[q] *Letter*, p. 14.

Perhaps, after all, the few passages in that work from which I am compelled to dissent, are more matters of language than of principle. Perhaps I may have given an interpretation to certain expressions which they were not intended to bear, or strained with over scrupulosity words which were not designed to express technical exactness. It is natural that a critic approaching a subject from the opposite side to that of the author he is criticising should be led to attach importance to the minuter features of objects lying immediately before him, which are overlooked from a more comprehensive point of view.

> "The critic eye, that microscope of wit,
> Sees hairs and pores, examines bit by bit."

But in this instance assuredly not without appreciating and admiring

"How parts relate to parts, or they to whole,
The body's harmony, the beaming soul."

The *Philosophy of the Inductive Sciences* may have many abler critics, but few more sincere admirers, than myself; and I shall consider the purpose of my animadversions fully accomplished, if they contribute in any degree to render the able and energetic protest of that work against sensationalism and positivism more effectual, by lessening its liability to even a verbal misrepresentation.

APPENDIX.

ON THE FUNDAMENTAL ASSUMPTIONS OF GEOMETRY.

THE Propositions which have been regarded by different writers as constituting the foundation of geometrical demonstration may be classified as follows:

I. Definitions, analysing the complex notions of the several magnitudes or figures.

II. Postulates, assuming the existence of the objects defined.

III. Axioms proper to Geometry, or synthetical judgments, stating self-evident properties of certain magnitudes.

IV. General Axioms, or analytical judgments, logically involving the notions of equality or inequality.

Some one or more of these, under various names, (for the language of the several writers has been by no means uniform,) have been selected at different times as the fundamental assumptions or premises from which the conclusions of Geometry may be demonstrated. A brief examination of each may perhaps help to clear the question.

I. According to Stewart, the properties of geometrical figures follow from the *Definitions* of those figures; the general axioms being mere barren truisms, and the axioms proper, (such as the 10th and 11th of Euclid,) being theorems requiring demonstration. In this theory, mathematical necessity becomes identified with logical, being only the result of the harmony of a process of thought with its original assumption. This consequence is accepted by Stewart himself, as well as by Archbishop Whately, who speaks of the denial of geometrical propositions as *self-contradictory.*

This view may be refuted either directly or by a *reductio ad absurdum;* for, firstly, it rests on an untenable assertion; secondly, it leads to an inadmissible consequence.

Firstly. If the properties of a figure follow from the definition of that figure, it must either be because they are implied in *some one attribute* of that definition, or because they are implied in *the whole.* A triangle, e. g. will have its angles equal to two right angles, either because it is a rectilineal figure, or because it is of three sides, or because it is both. The two first suppositions are manifestly false: the third begs the question; for why the notion of a triangle, regarded as a complex whole, has this property, is the very point at issue.

Hence it appears that the definitions of Geometry, so far as they are employed in demonstration, are merely nominal. From the analysis of the complex notion no conclusion is derived. The definition only serves to connect the notion as a whole with the name *triangle.* The question, why a rectilineal figure of three sides, be it called triangle or not, has its angles equal to two right angles, remains unanswered.

Secondly. If geometrical reasoning is merely "the logical filiation of consequences which follow from an assumed hypothesis," there is no reason why its conclusions should be more important than those of any other analysis of imaginary notions, such as (to use Mr. Mill's illustration) a deduction of the physiological properties of an imaginary animal, or the political history of an imaginary commonwealth. The whole character and history of mathematical science militates against the admission of this consequence.

II. Mr. Mill, while agreeing with Stewart that mathematical necessity is merely hypothetical and consequential, saw clearly that his doctrine concerning Definitions was untenable. This led him to adopt the second theory, according to which geometrical inferences depend on *Postulates* assuming the existence of the objects defined. Thus a triangle has its angles equal to two right angles because there may really exist a rectilineal figure having three sides; and this existence is implied, though not verbally expressed, in the definition.

This theory derives some apparent support from the use of the principle of superposition. When, for instance, the demonstration of the fourth proposition of Euclid supposes the triangle A B C to be applied to the triangle D E F, it clearly assumes the existence of both triangles, not merely as general notions, which are identical in thought, but as distinct individual magnitudes, occupying space, and capable of being transferred from one position in space to another. One non-entity cannot be applied to another. Thus far Mr. Mill's position is unquestionably true; but I think it may be shewn to be not itself the fundamental assumption of Geometry, but a consequence derivable from a higher assumption.

The existence is clearly only that which is implied in the possible construction of the figure. The actual or possible existence in nature of a body so figured, is not once appealed to in the demonstration, and might be denied without affecting its validity. The Postulate, therefore, implies the possible construction of a figure such as is contemplated in the proposition.

But this construction is mental, not manual. The figure as drawn upon paper is only a representative of the figure as imagined by the mind, and might be dispensed with altogether if the latter could be kept before us with sufficient steadiness. This brings us to Kant's principle of the possibility of mathematical science, viz. the power of *constructing* the objects of its concepts; i. e. of presenting them *a priori* in a pure intuition.

But how is this construction itself possible, and what conditions is it required to fulfil? Mr. Mill regards it as only possible *a posteriori*, and as subject to the same conditions as an object of sense. He says, "the points, lines, circles, and squares, which any one has in his mind, are simply copies of the points, lines, circles, and squares which he has known in his experience. We can reason about a line as if it had no breadth; but we cannot *conceive* a line without breadth." (Logic, b. ii. ch. v.) This is true; but the author is mistaken in supposing such a conception to be necessary to establish the *a priori* character of Mathematics. The true Postulate is not that of the possible existence of an object corresponding to the *definition*, but of one fulfilling the conditions of the *proper axiom*. We are not required to conceive a straight line as length without breadth: we are required to conceive it as such that two straight lines cannot enclose a space. The definition itself is but an imperfect attempt to describe in general terms what is known much more clearly by the image. It may serve to lead the thoughts of the learner to the proper image; but it was itself founded on a previous image in the mind of the teacher; and if the definition and the image differ, the former is in fault, not the latter.

III. This brings us to our third theory, which is that maintained by Kant. According to this theory, the fundamental assumptions of Geometry are *Proper Axioms*, or synthetical judgments *a priori;* and the possibility of forming such judgments depends on the power of constructing the objects to which they refer in a pure intuition, i. e. in an intuition containing no adventitious element external to the mind itself. The images of geometrical figures differ from all others in being not represented modifications of body, but presented modifications of space; and the universal validity of the synthetical judgment is a consequence of the universal presence of space as the form of every possible perception of body.

Three of these synthetical judgments are given in the 10th, 11th, and 12th axioms of Euclid, and either these or other axioms analogous to these must be assumed as evident by intuition, before any of the properties of more complex figures can be made known by demonstration. I do not say that Euclid has given the best and simplest forms of these axioms, but that in some form or other they are indispensable. To regard all such axioms as possibly demonstrable theorems is to be ignorant of the logical conditions under which demonstration is possible; for a synthetical judgment is demonstrable only on the condition that another synthetical judgment may be assumed. Οἱ *γὰρ ἁπάντων ζητοῦντες λόγον ἀναιροῦσι λόγον.*

It may be true that the image which gives rise to the intuitive perception of the axiom is not consciously contemplated as more perfect than the corresponding figure as seen in a body; but this does not prove that the axiom is really generalized from the latter. The inadequacy of sensible magnitudes for mathematical certainty does not arise from that of which we are immediately conscious, but from that of which we are not. The straight line as perceived is a quality of body; the straight line as imagined is a modification of space. The portions of the two actually presented at any time may not apparently differ from each other; but our empirical knowledge or ignorance of body may suggest actual or possible variations not perceived in the intuition; for the qualities of body have an objective existence independently of our perception, and therefore may or may not be adequately perceived at any one time. We see, for example, that a line running along the earth's surface is apparently straight; but we know that it is in reality an arc of the earth's curvature, and might be seen to be so in another position or with more acute organs. But the straight line in space exists only as imagined, and is imagined only as mathematically exact. The intuition, therefore, is adequate and valid for any extent of space, and in any portion. The apparent straightness of the visible line is the result of an imperfection in our bodily organs; and with more acute senses we might perceive its deviation. The presented straightness of the imaginary line results from the exactness of our constructive power; and a superior excellence in this would only enable us to extend the same image to a greater length, or to retain it more steadily before the mind.

IV. The synthetical axioms are thus the ground of all that is properly geometrical in our fundamental assumptions; but the analytical axioms are employed also, as expressing general conceptions of equality or inequality under which geometrical magnitudes may be brought. Stewart was led into his erroneous view of definitions by his contempt for the syllogism, which he would not allow to be under any circumstances the type of demonstrative reasoning. In this contempt Mr. Mill does not participate, and he has accordingly exhibited the fifth proposition of Euclid demonstrated in syllogistic form. In this demonstration we see both analytical and synthetical axioms employed as major premises; the former as general formulæ, founded on the conception of *equality;* the latter as the means of applying this general conception to geometrical magnitudes in which the test of equality is *coincidence.* One or the other will be employed in different syllogisms, according as the major term to be proved is *equality* or *coincidence.* The minor premises are furnished by the conditions, given or constructed, of the particular figure.

Against the form of the geometrical syllogism as exhibited by Mr. Mill the logician will have no objections to allege; though the metaphysician will not be disposed to acquiesce in his statement that the axioms of both kinds are gained by induction. And it is not strictly accurate to represent the first three axioms of Euclid as capable of proof by an imaginary superposition. To the axioms in their general form this principle is inapplicable; for coincidence is not the test of equality in general, but only of equality in superficial magnitudes. To the axioms as employed in Geometry the principle of superposition may be applied: but even here it adds nothing to their evidence. Magnitudes given as the sums of equal magnitudes are *ipso facto* thought as equal; and to have recourse to superposition tends to confound the evidence of logical necessity resting on the laws of thought, with that of geometrical necessity resting on the conditions of intuition.

Much error and confusion on this subject might have been avoided, had modern philosophers observed Aristotle's distinction between ἀρχαὶ ἐξ ὧν, or assumptions from which we reason, and ἀρχαὶ περὶ ὅ, or assumptions about the objects of our reasoning. In the former class he rightly places the axioms; in the latter, the definitions. But the true distinction between the axioms proper and the definitions, as synthetical and analytical judgments, has not, I think, been as yet accurately carried out in reference to Geometry.

Some interesting questions on the Logic of Mathematics still remain unanswered. But these must be postponed to another occasion.

THE END.

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